647510 **BSCmaMJ304x-6C** **Seat No :________**

B.Sc. Semester - 6 (NEP-2020) Examination
March/April-2026
MATH: Abstract Algebra -2 (Major-14)

Time: 2:00 Hours **Marks: 50**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two: (10)

(1) Define: Homomorphism of Groups.
Prove that a group homomorphism $\emptyset: G \to G'$ is one-to-one if and only if $Ker(\emptyset) = \{e\}$, where e =the identity element of the group G.

(2) State and prove the Cayley's theorem.

(3) State and prove the fundamental theorem of group homomorphism.

Que.2 Answer any two: (10)

(1) Define: Ring
Let R be a ring and let $a, b \in R$.
Then, prove that (i) $a0 = 0 = 0a$ (ii) $a(-b) = (-a)b = -(ab)$ (iii) $(-a)(-b) = ab$.

(2) Define: Integral Domain.
Prove that a finite integral domain is a field.

(3) Define: Boolean Ring.
Prove that the power set $\mathcal{P}(S)$ of a non-empty set S is a Boolean ring w.r.t. the following operations $+$ and $\cdot$
For $\forall\, A, B \in \mathcal{P}(S)$, $A + B = A \Delta B$ & $A \cdot B = A \cap B$.

Que.3 Answer any two: (10)

(1) For any two ideals I_1 and I_2 of a ring R, $I_1 \cup I_2$ is also an ideal of R *if and only if* either $I_1 \subset I_2$ or $I_2 \subset I_1$.

(2) Define: Simple ring.
Prove that the characteristic of a simple ring is either zero or a prime number.

(3) Define: Kernel of a ring homomorphism
Prove that a homomorphism $\emptyset: R \to R'$ is one-one if and only if $K_{\emptyset} = \{0\}$, where $K_{\emptyset}$= the kernel of $\emptyset$.

Que.4 Answer any two: (10)

(1) Define: Degree of a polynomial
For non-zero polynomials f & $g \in D[x]$, prove that $[fg] = [f] + [g]$.

(2) State and prove the division algorithm for polynomials.

(3) Define: Principal Ideal Domain.
Prove that $F[x]$ is a principal ideal domain.

Que.5 Answer any two: (10)

(1) Define: Isomorphism of groups.
Show that $(\mathbb{R}, +) \cong (\mathbb{R}^+, \cdot)$

(2) Define: Nilpotent element of a ring.
If a & b are nilpotent elements of a ring R and $ab = ba$ then prove that $(a + b)$ is also nilpotent.

(3) Find the g. c. d. of the polynomials
$f(x) = 6x^3 + 5x^2 - 2x + 25$ & $g(x) = 2x^2 - 3x + 5 \in \mathbb{R}[x]$ and express it in the form $a(x)f(x) + b(x)g(x)$.

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